B.Sc. Semester III (Honours) Examination, 2018-19

MATHEMATICS

Course ID : 32113 **Course Code : SHMTH-303C-7(T)**

Course Title : Numerical Models

Time: 1 Hour 15 Minutes **Full Marks: 25**

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

1. Answer *any five* questions: 1×5=5

(a) Find the percentage error in approximate representation of $^7/_6$ by $1{\cdot}16$.

(b) Round off the numbers 40·3586 and 0·0056812 to four significant digits.

(c) With usual notations, prove that $\Delta - \nabla \equiv \Delta\nabla$.

(d) Find the function $f(x)$ whose first difference is e^x.

(e) State the condition of convergence of the fixed point iteration method for finding a real root of the equation $f(x) = 0$.

(f) Using Euler's method, find *y*(0·05) given that $\frac{dy}{dx} = 1 + y^2, y(0) = 0$ and $h = 0{\cdot}05$.

(g) What is the geometrical significance of Trapezoidal rule for numerical integration?

(h) State Newton Gregory formula for forward interpolation.

2. Answer *any two* questions: 5×2=10

(a) What is interpolation? Using suitable interpolation formula, compute $f(1{\cdot}4)$and $f(3{\cdot}8)$. from the following data:

x	0	1	2	3	4
f(x)	1·0	1·5	2·2	3·1	4·2

5

(b) Explain the method of iteration for numerical solution of an equation of the form $x = \phi(x)$ and obtain the condition of convergence of the process. 5

(c) Establish the Newton-Cotes formula (closed type) in the form $I = (b-a)\sum_{i=0}^{n} H_i y_i$ for the integral $I = \int_a^b f(x)dx$, where H_i's are the Cote's coefficients and $y_i = f(x_i)$. Hence obtain the Trapezoidal formula for the given integral *I*. 4+1=5

(d) Given $\frac{dy}{dx} = x^3 + y, y(0) = 1$, compute $y(0{\cdot}02)$, by Euler's method correct upto four decimal places taking step length $h = 0{\cdot}01$. 5

3. Answer *any one* question: 10×1=10

(a) (i) The 'rate of convergence of an iterative formula is p'—What do you mean by this statement?

(ii) Show that the Newton-Raphson method has quadratic rate of convergence.

(iii) Determine $u(t)$ at $t = 0{\cdot}2$, $0{\cdot}4$ using the classical fourth order Runge–Kutta method, given that $u' = t/u$, $u(0) = 1$. 2+4+4=10

(b) (i) Describe Gauss-Seidal iterative method for the solution of a system of n linear equations in n unknowns. State the condition of convergence of this method. 4+1=5

(ii) Describe the power method for determining the largest eigen value in magnitude of a matrix A. (4+1)+5=10

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B.Sc. Semester III (Honours) Examination, 2018-19

MATHEMATICS

Course ID : 32114 **Course Code : SHMTH-304GE-3(T)**

Course Title : Algebra

Time: 2 Hours **Full Marks: 40**

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

1. Answer *any five* questions: 2×5=10

(a) Find the values of $i^{3/4}$.

(b) Apply Descartes' rule of signs to find the nature of the roots of the equation
$x^4 + 7x^3 + 2x - 1 = 0$.

(c) If *a, b, c* are positive reals, not all equal, then prove that
$(a + b + c)(bc + ca + ab) > 9abc$.

(d) Let *S* be the set of all lines in 3-space. A relation *R* is defined on *S* by "*lRm* if and only if *l* lies on the plane of *m*" for $l, m \in S$. Examine whether *R* is an equivalence relation.

(e) Let $f: \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = 3x^2 - 5$. Then find $f^{-1}(\{70\})$.

(f) Use Cayley-Hamilton theorem to find A^{-1} where $A = \begin{pmatrix} 3 & -2 \\ 4 & -3 \end{pmatrix}$.

(g) Find the dimension of the subspace S of $\mathbb{R}^4$,
where $S = \{(x, y, z, w) \in \mathbb{R}^4 : x + 2y - z + 2w = 0\}$.

(h) Examine whether the mapping $T: \mathbb{R}^3 \to \mathbb{R}^3$ defined by $T(x, y, z) = (x + y, y + z, x + 1)$ is linear or not? Justify your answer.

2. Answer *any four* questions: 5×4=20

(a) Solve: $x^3 - 9x + 28 = 0$. 5

(b) (i) If the roots of the equation $x^3 + ax^2 + bx + c = 0$ be in G.P, show that $b^3 = a^3c$.

(ii) Show that $3x^5 - 4x^2 + 6 = 0$ has at least two imaginary roots. 3+2=5

(c) (i) If x, y, z are positive real numbers and $x + y + z = 1$, prove that
$8xyz \leq (1 - x)(1 - y)(1 - z) \leq \frac{8}{27}$.

(ii) If n be a positive integer >1, prove that $\left(\frac{n+1}{2}\right)^n > n!$ 3+2=5

(d) The matrix of $T: \mathbb{R}^3 \to \mathbb{R}^3$ relative to the ordered basis $\{(0,1,1),(1,0,1),(1,1,0)\}$ is $\begin{pmatrix} 0 & 3 & 0 \\ 2 & 3 & -2 \\ 2 & -1 & 2 \end{pmatrix}$. Find the linear transformation *T*. Find the matrix of *T* relative to the ordered basis $\{(2,1,1),(1,2,1),(1,1,2)\}$. 3+2=5

(e) (i) Use the theory of Congruences to prove that $17 | (2^{3n+1} + 3 \cdot 5^{2n+1})$ for all positive integers *n*.

(ii) Let *a, b, c, m* be positive integers such that $ac \equiv bc \pmod{m}$ and $\gcd(c,m) = 1$ then prove that $a \equiv b \pmod{m}$. 3+2=5

(f) Solve if possible, the system of equations 5

$$x_1 + 2x_2 - x_3 = 0$$
$$-x_1 + x_2 + 2x_3 = 2$$
$$2x_1 + x_2 - 3x_3 = 2$$

3. Answer *any one* question: 10×1=10

(a) (i) If α, β are the roots of the equation $x^2 - 2x + 4 = 0$, prove that

$$\alpha^n + \beta^n = 2^{n+1} cos \frac{n\pi}{3}.$$

(ii) Show that the sum of the 99th power of the roots of the equation $x^5 = 1$ is zero.

(iii) Verify Cayley-Hamilton theorem for the matrix $\begin{pmatrix} 1 & 0 & 2 \\ 0 & -1 & 1 \\ 0 & 1 & 0 \end{pmatrix}$. Hence find A^{-1}.

3+3+4=10

(b) (i) Apply elementary row operations to reduce the following matrix to a row echelon matrix and find the rank of the matrix $\begin{pmatrix} 2 & 0 & 4 & 2 \\ 3 & 2 & 6 & 5 \\ 5 & 2 & 10 & 7 \\ 0 & 3 & 2 & 5 \end{pmatrix}$.

(ii) State first principle of mathematical induction. Apply this to prove that $3^{2n} = 8n - 1$ is divisible by 64. (4+1)+(2+3)=10

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B.Sc. Semester III (Honours) Examination, 2018-19

MATHEMATICS

Course ID : 32115 **Course Code : SHMTH-305SEC-1(T)**

Course Title : C Programming Language (New)

Time: 2 Hours **Full Marks: 40**

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

1. Answer *any five* questions: 2×5=10

(a) Define source and object programs.

(b) If K is an integer variable and a is a real variable then what will be the value of $K = {}^{2}/_{9.0}$ and $a = {}^{2}/_{9.0}$.

(c) What is the difference between *abs* ( ) and *fabs* ( ) functions?

(d) Consider the following 'C' statement: $X = (j + k > 5)?\,(j + k): 5$; what will happen when this statement is executed if (i) $j = 5$ and $k = 3$ and $(ii) j = 1$ and $k = -3$?

(e) State two differences between a compiler and an interpreter.

(f) Describe the difference between = and = = symbols in C programming.

(g) If the matrix $\begin{bmatrix} 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 7 \\ 3 & 6 & 2 & 9 \end{bmatrix}$ is declared as a two-dimensional array variable, $a[3][4]$, then find the values of $a[1][3]$ and $a[2][1]$.

(h) Find the output of the following program segment:

$int\ sum\ =\ 0, i;$

$for\ (i = 0; i <= 5;\ i++)$

$sum = sum + i\,;$

$printf($"%d %d"$, sum, i);$

2. Answer *any four* questions: 5×4=20

(a) (i) What is the machine language? State its main advantage and disadvantage.

(ii) What do you mean by problem oriented computer language? Explain with example.

1+(1+1)+2=5

(b) Write a program to find factorial of a number. 5

(c) Find a C program to calculate the value of ${}^{n}p_r$. 5

(d) Write a C-program to find the root and their nature of a given quadratic equation $ax^2 + bx + c = 0$. 5

(e) What would be the output of the following program segment:

```
main ( )
{
    int i = 2, j = 3, k, l;
    float a, b;
    k = i/j*j;
    l = j/i*i;
    a = i/j*j;
    b = j/i * i;
    printf("%d%d%f%f", k, l, a, b);
}
```

5

Necessary calculations should be mentioned clearly.

(f) (i) How the array variables are declared in 'C'? Illustrate the initialization of an one-dimensional array with example.

(ii) Write a short note on 'while' loop in 'C'. 3+2=5

3. Answer *any one* questions: 10×1=10

(a) (i) Write a complete 'C'-program to find the sum and average of 100 real numbers by using subscripted variables. (Input should be given by using *scanf*( ) function)

(ii) Determine the value of the following logical expression

if $a = 5, b = 15$ and $c = -7$ $b > 15$ && $c < 0 || a > 0$ && $(a + b + c) > 0$.

(Show the intermediate logical calculations).

(iii) Write a short note on 'shorthand assignment operator'. 6+2+2=10

(b) Write a program to print all prime numbers from 1 to 300 using nested loops, break statements, continue statement. What will be the output of the following program?

```
#include < stdio.h >
int main ( )
{
    int x = 10, y = 20;
    if (x > = 2 and y < = 50)
    print ("%d\n", x);
    }
```

7+3=10

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B.Sc. Semester III (Honours) Practical Examination, 2018-19

MATHEMATICS

Course ID : 32321 **Course Code : SHMTH-303C-7(P)**

Course Title : Numerical Models Lab

Time: 2 Hours **Full Marks: 15**

Candidates are required to give their answers in their own words as far as practicable.

SET - 1

Candidates are required to answer one question (indicated in the card drawn by the candidate) from Group-A and one question (also indicated in the card) from Group-B. They are also required to write the suitable programs in C programming language to solve the problems.

Marks distribution:

For the Problem from Group-A — 5 marks

For the Problem form Group-B — 3 marks

Sessional — 4 marks

Viva voce — 3 marks

Group-A

(Numerical Models)

1. Using Newton-Raphson's method, find one positive root of the equation:

$$X\log_{10} X - (J+6)X^4 + 35.2 = 0,$$

correct up to five places of decimals, the value of J being given in the card drawn by you. The output should contain the initial approximation, toleration, the actual number of iteration taken and the required root.

2. Using the following Table, find the value of Y correct up to five places of decimals when $X = 1 \cdot 147$ by Lagrange's interpolation method, the value of J being given in the card drawn by you. The output should contain the number of points, the value of X for which Y is to be calculated, the values of X and Y given in the table and the required result.

Table–1

X	Y
1·12	1·81 × J
1·14	1·77 × J
1·16	1·73 × J
1·18	1·70 × J
1·20	1·67 × J
1·22	1·64 × J

3. Using the Trapezoidal rule, find the value of the integral, taking 50 equal sub-intervals:

$$\int_0^1 \frac{e^X + \frac{2X^2}{5} + \sin(4X + \frac{13}{J})}{2X + \sqrt{JX^2 + 2X + 17.1}}\,dX$$

Correct up to 6 places of decimals, the value of *J* being given in the card drawn by you. The output should contain the limits of integration, number of sub-intervals, the lengths of the sub-intervals and the value of the integral.

4. Using the Simpson's 1/3 rule, find the value of the integral, taking 50 equal sub-intervals:

$$\int_0^{\frac{\pi}{2}} \sqrt{J - 0.162\sin^2 X\,dX}$$

Correct up to 6 places of decimals, where *J* is given in the card drawn by you. The output should contain the limits of integration, number of sub-intervals, the lengths of sub-intervals and the value of the integral.

5. Using the Fourth Order Runge-Kutta method, find the values of *Y* at *X* = 1·1, 1·2, 1·3, 1·4, 1·5, 1·6, 1·7, 1·8, 1·9, 2·0 taking X_0=1·0, Y_0=1·0, *h*=0·1 from the following differential equation:

$$\frac{dY}{dX} = \frac{2X + 2Y^2 + \sin(X^2 + 0.3Y + \frac{J}{4.5})}{2Y + \sqrt{4.9X^2 + JY + 12.1}}$$

Correct up to 6 places of decimals, the value of *J* being given in the card drawn by you. The output should contain the values of X_0, Y_0, and the required result.

6. Using the Power method find the largest Eigenvalue and the corresponding Eigenvector correct up to five places of decimals from the matrix $\begin{pmatrix} 7.22 & 6.1 + J \\ 3.57 & 5.77 \end{pmatrix}$, the value of *J* being given in the card drawn by you.

Group-B

(Unknown Problems)

1. Find the sum of the following series correct up to five places of decimals: $\sum_{n=1}^{J+8} \frac{1}{n^2}$, the value of *J* being given in the card drawn by you.

2. Find the largest number of the given set: {4, 5, 7, 1, 3, 8, 11, 23, *J*, 12, 34} using array.

3. Calculate the value of the expression at the point x=2·5 correct up to five places of decimals $e^x + \sqrt{Jx^2 + 3.5x + 5.7}$, the value of *J* being given in your card.

4. Find the value of the following by using function subprogram for *x* = *J*+2, *y* = *J*/5 correct up to five places of decimals: $f(x, y) = x^2 + y^2$, the value of J being given in your card.

5. Calculate the surface area of a cylinder with radius = (4·2 + *J*) cm and height = 3·8 cm.

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B.Sc. Semester III (Honours) Practical Examination, 2018-19

MATHEMATICS

Course ID : 32321 **Course Code : SHMTH-303C-7(P)**

Course Title : Numerical Models Lab

Time: 2 Hours **Full Marks: 15**

Candidates are required to give their answers in their own words as far as practicable.

SET - 2

Candidates are required to answer one question (indicated in the card drawn by the candidate) from Group-A and one question (also indicated in the card) from Group-B. They are also required to write the suitable programs in C programming language to solve the problems.

Marks distribution:

For the Problem from Group-A — 5 marks

For the Problem from Group-B — 3 marks

Sessional — 4 marks

Viva voce — 3 marks

Group-A

(Numerical Models)

1. Using Newton-Raphson's method, find one positive root of the equation:

$$2.12X^3 + 1.1X - \sin\left(\frac{X}{9.1} + \frac{J}{3.3}\right) = 0$$

Correct up to five places of decimals, the value of J being given in the card drawn by you. The output should contain the initial approximation, error toleration, the actual number of iterations taken and the required root.

2. Using the following Table, find the value of Y correct up to five places of decimals when $X = 1 \cdot 282$ by Lagrange's interpolation method, the value of J being given in the card drawn by you. The output should contain the number of points, the value of X for which Y is to be calculated, the values of X and Y given in the table and the required result.

Table

X	Y
1·22	1·64×J
1·24	1·60×J
1·26	1·57×J
1·28	1·54×J
1·30	1·51×J
1·32	1·49×J

3. Using the Trapezoidal rule, find the value of the integral, taking 50 equal sub-intervals:

$$\int_{1}^{2} \frac{4.12X^2 + \log_e(2X^2 + 5X + J)}{2X + \sqrt{X^2 + 5.2X + 4}}\, dX$$

Correct up to 6 places of decimals, the value of *J* being given in the card drawn by you. The output should contain the limits of integration, number of sub-intervals, the lengths of the sub-intervals and the value of the integral.

4. Using the Simpson's 1/3 rule, find the value of the integral, taking 50 equal sub-intervals:

$$\int_{1}^{1.5} \{JX^2 + (\log_e(3.5X + J)^2\} dX$$

Correct up to 6 places of decimal, the value of *J* being given in the card drawn by you. The output should contain the limits of integration, number of sub-intervals, the lengths of sub-intervals and the value of the integral.

5. Using the Fourth Order Runge-Kutta method, find the values of *Y* at X= 1·1, 1·2, 1·3, 1·4, 1·5, 1·6, 1·7, 1·8, 1·9, 2·0 taking X_0=1·0, Y_0=2·5, *h*=0·1 from the following differential equation:

$$\frac{dY}{dX} = XY - \frac{J}{10}Y^2 + \log_e\left(12.1Y^2 + 4.5X + \frac{J}{7}\right)$$

Correct up to 6 places of decimals, the value of *J* being given in the card drawn by you. The output should contain the values of X_0, Y_0 h and the required result.

6. Using the Power method find the largest Eigenvalue and the corresponding Eigenvector correct up to five places of decimals from the matrix $\begin{pmatrix} 1.512 & J \\ 3.27 & 8.37 \end{pmatrix}$, the value of *J* being given in the card drawn by you.

Group-B

(Unknown Problems)

1. Find the sum of the following series correct up to five places of decimals: $\sum_{n=1}^{J+5} \frac{1}{n!}$, the value of *J* being given in the card drawn by you.

2. Find the least number of the given set: {4, 5, 7, *J*, 3, 8, 1, 23, 12, 34} using array.

3. Calculate the value of the expression at the point $x = 2.5, \alpha = \pi/4$ correct up to five places of decimals $\sin\alpha + \sqrt{Jx^2 + 3.5x + 5.7}$, the value of *J* being given in your card.

4. Find the value of the following function by using function subprogram for *x*=*J*+2, *y*=*J*/5 correct up to five places of decimals: $f(x, y) = x^2 + y^2 + 5xy$, the value of *J* being given in your card.

5. Calculate the volume of a sphere with radius = (4·2+*J*) cm.

B.Sc. Semester III (Honours) Practical Examination, 2018-19

MATHEMATICS

Course ID : 32321 **Course Code : SHMTH-303C-7(P)**

Course Title : Numerical Models Lab

Time: 2 Hours **Full Marks: 15**

Candidates are required to give their answers in their own words as far as practicable.

SET - 3

Candidates are required to answer one question (indicated in the card drawn by the candidate) from Group-A and one question (also indicated in the card) from Group-B. They are also required to write the suitable programs in C programming language to solve the problems.

Marks distribution:

For the Problem from Group-A — 5 marks

For the Problem from Group-B — 3 marks

Sessional — 4 marks

Viva voce — 3 marks

Group-A

(Numerical Models)

1. Using Newton-Raphson's method, find one positive root of the equation:

$$(J+1)^X + X - 3.99 = 0$$

Correct up to five places of decimals, the value of J being given in the card drawn by you. The output should contain the initial approximation, toleration, the actual number of iterations taken and the required root.

2. Using the following Table, find the value of Y correct up to five places of decimals when $X = 1 \cdot 399$ by Lagrange's interpolation method, the value of J being given in the card drawn by you. The output should contain the number of points, the value of X for which Y is to be calculated, the values of X and Y given in the table and the required result.

Table

X	Y
1·32	1·49×J
1·34	1·46×J
1·36	1·43×J
1·38	1·40×J
1·40	1·38×J
1·42	1·36×J

3. Using the Trapezoidal rule, find the value of the integral, taking 50 equal sub-intervals:

$$\int_{0.5}^{1.5} \frac{1.5+\cos(7.5X+J)}{J+\sqrt{X^2+X+2.5}}\,dX$$

Correct up to 6 places of decimals, the value of *J* being given in the card drawn by you. The output should contain the limits of integration, number of sub-intervals, the lengths of sub-intervals and the value of the integral.

4. Using the Simpson's 1/3 rule, find the value of the integral, taking 50 equal sub-intervals:

$$\int_{1}^{2} \left\{\log_e(X)+\sqrt{5X^2+\frac{5X}{J}+13}\right\} dX$$

Correct up to 6 places of decimals, the value of *J* being given in the card drawn by you. The output should contain the limits of integration, number of sub-intervals, the lengths of sub-intervals and the value of the integral.

5. Using the Fourth Order Runge-Kutta method, find the values of *Y* at *X*= 1·1, 1·2, 1·3, 1·4, 1·5, 1·6, 1·7, 1·8, 1·9, 2·0 taking X_0=1·0, Y_0=1·0, *h*=0·1 from the following differential equation:

$$\frac{dY}{dX} = XY - \frac{J}{10}Y^2 + \sqrt[3]{X^2+J}$$

Correct up to 6 places of decimals, the value of *J* being given in the card drawn by you. The output should contain the values of X_0, Y_0, h and the required result.

6. Using the Power method find the largest Eigenvalue and the corresponding Eigenvector correct up to five places of decimals from the matrix $\begin{pmatrix} 8.41 & J+0.02 \\ 3.27 & 1.5 \end{pmatrix}$, the value of *J* being given in the card drawn by you.

Group-B

(Unknown Problems)

1. Find the sum of the following series correct up to five places of decimals: $\sum_{n=1}^{J+8}\frac{1}{n+1}$, the value of *J* being given in the card drawn by you.

2. Find the largest number of the given set: {4, 5, 7, 1, 3, 7, 11, 23, *J*, 12, 34} using array.

3. Calculate the value of the expression for $a=J, b=2, x=0.5,$ correct up to five places of decimals $a^3+b^3+e^{(x^2+2b)}$, the value of *J* being given in your card.

4. Find the value of the following function by using function subprogram for *x*=*J*+2, θ=45° correct up to five places of decimals: $f(x)=\sqrt{x^2+2x+\cos\theta}$, the value of *J* being given in your card.

5. Calculate the volume of a right circular cone with radius of the base is (4·2+*J*) cm and height = 3·8 cm.

B.Sc. Semester III (Honours) Practical Examination, 2018-19

MATHEMATICS

Course ID : 32321 **Course Code : SHMTH-303C-7(P)**

Course Title : Numerical Models Lab

Time: 2 Hours **Full Marks: 15**

Candidates are required to give their answers in their own words as far as practicable.

SET - 4

Candidates are required to answer one question (indicated in the card drawn by the candidate) from Group-A and one question (also indicated in the card) from Group-B. They are also required to write the suitable programs in C programming language to solve the problems.

Marks distribution:

For the Problem from Group-A — 5 marks

For the Problem from Group-B — 3 marks

Sessional — 4 marks

Viva voce — 3 marks

Group-A

(Numerical Models)

1. Using Newton-Raphson's method, find one positive root of the equation:

$$X^3 - 3\cos\left(\frac{5X}{9} + \frac{J}{3}\right) + 4.5X - 1.2 = 0$$

Correct up to five places of decimals, the value of J being given in the card drawn by you. The output should contain the initial approximation, toleration, the actual number of iterations taken and the required root.

2. Using the following Table, find the value of Y correct up to five places of decimals when $X = 1\cdot 45$ by Lagrange's interpolation method, the value of J being given in the card drawn by you. The output should contain the number of points, the value of X for which Y is to be calculated, the values of X and Y given in the table and the required result.

Table

X	Y
1·42	1·36×J
1·44	1·33×J
1·46	1·31×J
1·48	1·29×J
1·50	1·27×J
1·52	1·25×J

3. Using the Trapezoidal rule, find the value of the integral, taking 50 equal sub-intervals:

$$\int_{2.5}^{3.5} \frac{\log_e X \sqrt{2X^2 + \frac{5X}{J} + 13}}{2X^2 + JX + \sqrt{X^3 + X^2 + JX + 1}}\, dX$$

Correct up to 6 places of decimals, the value of *J* being given in the card drawn by you. The output should contain the limits of integration, number of sub-intervals, the lengths of sub-intervals and the value of the integral.

4. Using the Simpson's 1/3 rule, find the value of the integral, taking 50 equal sub-intervals:

$$\int_{0.5}^{1.5} \frac{J\sqrt{X+1}}{\sqrt{2\pi}} e^{-\frac{X^2}{2}}\, dX$$

Correct up to 6 places of decimals, the value of *J* being given in the card drawn by you. The output should contain the limits of integration, number of sub-intervals, the lengths of sub-intervals and the value of the integral.

5. Using the Fourth Order Runge-Kutta method, find the values of Y at X= 1·1, 1·2, 1·3, 1·4, 1·5, 1·6, 1·7, 1·8, 1·9, 2·0 taking X_0=1·0, Y_0=1·0, *h*=0·1 from the following differential equation:

$$\frac{dY}{dX} = 2Y + \sqrt{4.9X^2 + JY + 12.1}$$

Correct up to 6 places of decimals, the value of *J* being given in the card drawn by you. The output should contain the values of X_0, Y_0, h and the required result.

6. Using the Power method find the largest Eigenvalue and the corresponding Eigenvector correct up to five places of decimals from the matrix $\begin{pmatrix} 0.92 & J+1 \\ 3.25 & 2.202 \end{pmatrix}$, the value of *J* being given in the card drawn by you.

Group-B

(Unknown Problems)

1. Find the sum of the following series correct up to five places of decimals: $\sum_{n=1}^{J+5} \frac{n+1}{n^3}$, the value of *J* being given in the card drawn by you.

2. Find the least number of the given set: {4, 5, 7, 3, *J*, 8, 11, 23, 12, 34} using array.

3. Calculate the value of the expression at the point $x = 2.5$ correct up to five places of decimals $\log_{10} x + \sqrt{Jx^2 + 3.5x + 5.7}$, the value of *J* being given in your card.

4. Find the value of the following function by using function subprogram for *x*=*J*+2, *y*=*J*/5 correct up to five places of decimals: $f(x, y) = \sqrt[3]{x^2 + y^2}$, the value of *J* being given in your card.

5. Calculate the surface area of a cylinder with radius = (3·2+*J*) cm and height = 4 cm.

B.Sc. Semester III (Honours) Practical Examination, 2018-19

MATHEMATICS

Course ID : 32321 **Course Code : SHMTH-303C-7(P)**

Course Title : Numerical Models Lab

Time: 2 Hours **Full Marks: 15**

Candidates are required to give their answers in their own words as far as practicable.

SET - 5

Candidates are required to answer one question (indicated in the card drawn by the candidate) from Group-A and one question (also indicated in the card) from Group-B. They are also required to write the suitable programs in C programming language to solve the problems.

Marks distribution:

For the Problem from Group-A — 5 marks

For the Problem from Group-B — 3 marks

Sessional — 4 marks

Viva voce — 3 marks

Group-A

(Numerical Models)

1. Using Newton-Raphson's method, find one positive root of the equation:

$$X = J + 1 - 2^X$$

correct up to five places of decimals, the value of J being given in the card drawn by you. The output should contain the initial approximation, toleration, the actual number of iteration taken and the required root.

2. Using the following Table, find the value of Y correct up to five places of decimals when $X = 1 \cdot 175$ by Lagrange's interpolation method, the value of J being given in the card drawn by you. The output should contain the number of points, the value of X for which Y is to be calculate, the values of X and Y given in the table and the required result.

Table

X	Y
1·12	2·18×J
1·14	2·20×J
1·16	2·22×J
1·18	2·24×J
1·20	2·26×J
1·22	2·28×J

3. Using the Trapezoidal rule, find the value of the integral, taking 50 equal sub-intervals:

$$\int_0^2 \frac{7X^2 + \sin\left(3X^2 + 1.75X + \frac{J}{5.1}\right)}{3X + \sqrt{2.9X^2 + X + 11}}\, dX$$

correct up to 6 places of decimals, the value of *J* being given in the card drawn by you. The output should contain the limits of integration, number of sub-intervals, the lengths of sub-intervals and the value of the integral.

4. Using the Simpson's 1/3 rule, find the value of the integral, taking 50 equal sub-intervals:

$$\int_0^{\frac{\pi}{2}} \sqrt{J(\theta + 7) - 0.16\sin^2\theta\, d\theta}$$

correct up to 6 places of decimal, the value of *J* being given in the card drawn by you. The output should contain the limits of integration, number of sub-intervals, the lengths of sub-intervals and the value of the integral.

5. Using the Fourth Order Runge-Kutta method, find the values of *Y* at X= 1·1, 1·2, 1·3, 1·4, 1·5, 1·6, 1·7, 1·8, 1·9, 2·0 taking X_0=1·0, Y_0=1·0, h=0·1 from the following differential equation

$$\frac{dY}{dX} = \frac{\sqrt{X + Y}}{2.5 + \sin\left(12Y^2 + 4.5X + \frac{J}{7.1}\right)}$$

correct up to 6 places of decimals, the value of *J* being given in the card drawn by you. The output should contain the values of X_0, Y_0 h and the required result.

6. Using the Power method find the largest Eigenvalue and the corresponding Eigenvector correct up to five places of decimals from the matrix $\begin{pmatrix} 2.202 & 3.25 \\ 1 + J & 0.92 \end{pmatrix}$, the value of *J* being given in the card drawn by you.

Group-B

(Unknown Problems)

1. Find the sum of the following series correct up to five places of decimals:
$\sum_{n=1}^{J+5} \frac{n+3}{n^2}$, the value of *J* being given in the card drawn by you.

2. Find the largest number of the given set: {4, 5, 7, 1, 3, 8, 11, 23, *J*, 12, 34} using array.

3. Calculate the value of the expression at the point $x = 2.5$ correct up to five places of decimals $e^x + x^x + \sqrt{Jx^2 + 3.5x + 5.7}$, the value of *J* being given in your card.

4. Find the value of the following function by using function subprogram for *x*=*J*+2, *y*=*J*/5 correct up to five places of decimals: $f(x, y) = x^2 + y^2 + 3(x + y)$, the value of *J* being given in your card.

5. Calculate the volume of a cylinder with radius = (4·2+*J*) cm and height = 3·8 cm.

B.Sc. Semester III (Honours) Practical Examination, 2018-19

MATHEMATICS

Course ID : 32321 **Course Code : SHMTH-303C-7(P)**

Course Title : Numerical Models Lab

Time: 2 Hours **Full Marks: 15**

Candidates are required to give their answers in their own words as far as practicable.

SET - 6

Candidates are required to answer one question (indicated in the card drawn by the candidate) from Group-A and one question (also indicated in the card) from Group-B. They are also required to write the suitable programs in C programming language to solve the problems.

Marks distribution:

For the Problem from Group-A — 5 marks

For the Problem form Group-B — 3 marks

Sessional — 4 marks

Viva voce — 3 marks

Group-A

(Numerical Models)

1. Using Newton-Raphson's method, find one positive root of the equation:

$$1.19X\log_{10} X-(J+6.05)X^4+35=0$$

correct up to five places of decimals, the value of J being given in the card drawn by you. The output should contain the initial approximation, toleration, the actual number of iteration taken and the required root.

2. Using the following Table, find the value of Y correct up to five places of decimals when $X=2\cdot 311$ by Lagrange's interpolation method, the value of J being given in the card drawn by you. The output should contain the number of points, the value of X for which Y is to be calculated, the values of X and Y given in the table and the required result.

Table

X	Y
2·22	3·28×J
2·24	3·30×J
2·26	3·32×J
2·28	3·34×J
2·30	3·36×J
2·32	3·37×J

3. Using the Trapezoidal rule, find the value of the integral, taking 50 equal sub-intervals:

$$\int_0^1 \frac{e^2 + \frac{4X^2}{5} + \sin(4X + \frac{13}{J})}{4X + \sqrt{JX^2 + 4X + 17}} dX$$

correct up to 6 places of decimals, the value of *J* being given in the card drawn by you. The output should contain the limits of integration, number of sub-intervals, the lengths of sub-intervals and the value of the integral.

4. Using the Simpson's 1/3 rule, find the value of the integral, taking 50 equal sub-intervals:

$$\int_0^{0.5} \sqrt{\frac{J - 0.75\theta^2}{1 - \theta^2}} + \sin(J\theta) d\theta$$

correct up to 6 places of decimals, the value of *J* being given in the card drawn by you. The output should contain the limits of integration, number of sub-intervals, the lengths of sub-intervals and the value of the integral.

5. Using the Fourth Order Runge-Kutta method, find the values of *Y* at X= 1·1, 1·2, 1·3, 1·4, 1·5, 1·6, 1·7, 1·8, 1·9, 2·0 taking X_0=1·0, Y_0=1·0, h=0·1 from the following differential equation

$$\frac{dY}{dX} = \frac{5.5X^2 + 5Y^2 + \log_e\left(X + 2.7Y + \frac{J}{2.5}\right)}{5X + \sqrt{JX + 3.4Y^2 + 10.5}}$$

correct up to 6 places of decimals, the value of *J* being given in the card drawn by you. The output should contain the values of X_0, Y_0 h and the required result.

6. Using the Power method find the largest Eigenvalue and the corresponding Eigenvector correct up to five places of decimals from the matrix $\begin{pmatrix} -2.01 & 3.932 \\ 2 + J & 4.01 \end{pmatrix}$, the value of *J* being given in the card drawn by you.

Group-B

(Unknown Problems)

1. Find the sum of the following series correct up to five places of decimals: $\sum_{n=1}^{J+8} \frac{n+5}{n^2+7}$, the value of *J* being given in the card drawn by you.

2. Find the least number of the given set: {4, 5, 7, *J*, 3, 8, 11, 23, 12, 34} using array.

3. Calculate the value of the expression at the point $x = 2.5, \alpha = \beta = \gamma = 7°$ correct up to five places of decimals $\sqrt{Jx^2 + 3.5x + 5.7} + \sin(\alpha + \beta + \gamma)$, the value of *J* being given in your card.

4. Find the value of the following function by using function subprogram for *x*=*J*+2, *y*=*J*/5 correct up to five places of decimals: $f(x, y) = x^2 + y^2 \sec(x + y)$, the value of *J* being given in your card.

5. Calculate the volume and surface area of a cylinder with radius = (4·2+*J*) cm and height = 3·8 cm.

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B.Sc. Semester III (General) Examination, 2018-19

MATHEMATICS

Course ID : 32118 **Course Code : SPMTH-301C-1C(T)**

Course Title : Algebra

Time: 2 Hours **Full Marks: 40**

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

1. Answer *any five* questions: 2×5=10

(a) Find the modulus and amplitude of $\frac{3+5i}{2-3i}$.

(b) If *a, b, c* be positive real numbers, prove that
$(a^2b + b^2c + c^2a)(ab^2 + bc^2 + ca^2) \geq 9a^2b^2c^2$.

(c) If α be a multiple root of order 3 of the equation $x^4 + bx^2 + cx + d = 0$ $(d \neq 0)$ show that $\alpha = -\frac{8d}{3c}$.

(d) A relation R is defined on the set $\mathbb{Z}$ by "aRb if and only if $ab > 0$" for $a, b \in \mathbb{Z}$. Examine if R is an equivalence relation.

(e) Find the dimension of the subspace S of $\mathbb{R}^3$ where $S = \{(x, y, z): 2x + y - z = 0\}$.

(f) Let $f, g: \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = 3x^2 - 5$ *and* $g(x) = \frac{x}{x^2+1}$. Then find *f*o*g* and *g*o*f*.

(g) Find *x* such that the rank of $A = \begin{pmatrix} 2 & 1 & 4 \\ 1 & x & 2 \\ 4 & 0 & x+2 \end{pmatrix}$ is 2.

(h) Verify Cayley-Hamilton theorem for $A = \begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}$.

2. Answer *any four* questions: 5×4=20

(a) If *a, b, c, d* be distinct positive real numbers and $S = a + b + c + d$ then prove that
$\frac{S}{S-a} + \frac{S}{S-b} + \frac{S}{S-c} + \frac{S}{S-d} > 5\frac{1}{3}$.

(b) For what values of *K*, the following equations
$x + y + z = 1, 2x + y + 4z = K, 4x + y + 10z = K^2$, have solutions and completely in each case.

(c) Solve the equation $2x^4 - 5x^3 - 15x^2 + 10x + 8 = 0$, the roots being in geometric progression.

(d) (i) If *a* is prime to *b*, prove that $a + b$ is prime to *ab*.

(ii) Prove that the product of any three consecutive integers is divisible by 6. 2+3=5

(e) (i) Show that $\mathbb{N}$ and $\mathbb{Z}$ have the same cardinality.

(ii) Let *A, B* be both finite sets of *n* elements and a mapping $f: A \to B$ is injective. Prove that *f* is a bijection. 3+2=5

(f) Find the eigenvalues and the corresponding eigen vectors of the matrix

$$\begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}.$$

3. Answer *any one* question: 10×1=10

(a) (i) Solve : $x^3 - 12x + 65 = 0$

(ii) Use the theory of Congruences to show that $7|(2^{5n+3} + 5^{2n+3})$ for all positive integer *n*.

(iii) Apply Descartes' rule of signs to find the nature of the roots of the equation $2x^4 + 14x^2 + 7x - 8 = 0$ 4+3+3=10

(b) (i) Verify Cayley-Hamilton theorem for $A = \begin{pmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{pmatrix}$ and find A^{-1}. 2+3=5

(ii) State first principle of induction and using this principle prove that $2^n < n!$ for $n \in \mathbb{N}$ and $n \geq 4$. 2+3=5

B.Sc. Semester III (General) Examination, 2018-19

MATHEMATICS

Course ID : 32110 **Course Code : SPMTH-304SEC-1(T)**

Course Title : Logic and Sets

Time: 2 Hours **Full Marks: 40**

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

1. Answer *any five* questions: 5×2=10

(a) If $A = \{3, 4, 5, 7, 9\}, B = \{5, 9, 1, 6\}$ and $C = \{3, 2\}$, find all elements of the set $(A\Delta B) \times C$, when $\Delta=$ symmetric difference of two sets.

(b) List the elements of $A = \{x : x \in \mathbb{N}, 4 + x = 3\}$. Where $\mathbb{N}$ is the set of natural number.

(c) If $A = \{2, 12, 32\}$ and $B = \{1, 4, 8, 16\}$ and the universal set $\cup = \{1, 2, 4, 8, 12, 16, 32\}$ then prove that $(A \cap B)' = A' \cup B'$ where A' represents the complement of A.

(d) If $n(A) = 20,\ n(B) = 35$ and $n(A \cup B) = 45$ by drawing Venn-Euler diagram. Show that $n(A \cap B) = 10$.

(e) Write down the elements of the power set of the set $X = \{x, y, z, w\}$.

(f) Find the truth table of the conjunction of two statements *p* and *q*.

(g) Find the truth table of the disjunction of two statements *p* and *q*.

(h) If $A = \{1, 2\}, B = \{2, 3\}, C = \{3, 4\}$, find $A \times (B \cup C)$.

2. Answer *any four* questions: 5×4=20

(a) If A, B, C are subsets of the universal set *X*, prove the following:

(i) $(A \cap B) \cup (A \cap B') \cup (A' \cap B) \cup (A' \cap B') = X$

(ii) $(A \cup B \cup C') \cap (A \cup B' \cup C') = A \cup C'$. 3+2=5

(b) If A, B, C are subsets of an universal set S. Prove that $A \times (B \cup C) = (A \times B) \cup (A \times C)$.

(c) If $n(A) = 100, n(B) = 90, n(C) = 100, n(A \cap B) = 60, n(B \cap C) = 40, n(A \cap C) = 45$, $n(A \cup B \cup C) = 200$, find $n(A \cap B \cap C)$.

(d) Let p, q, r be statements. Then show that distributive law,

$p \vee (q \wedge r) = (p \vee q) \wedge (p \vee r)$ holds by truth table.

(e) Find whether the relations R_1 and R_2 as defined below in the set $A = \{1, 2, 3\}$ are

(i) reflexive, (ii) symmetric, (iii) transitive.

(a) $R_1 = \{(2, 1), (1, 2), (3, 3)\}$

(b) $R_2 = \{(3, 3)\}$ 3+2=5

(f) If R be a relation in the set of integers $\mathbb{Z}$ defined by $R = \{(x, y) : x \in \mathbb{Z}, y \in \mathbb{Z}, (x - y)$ is divisible by $6\}$ then prove that R is equivalence relation. Find all the distinct equivalence classes of the relation R.

3. Answer *any one* question: 10×1=10

(a) (i) Let $\mathbb{Z}$ be the set of all integers and A, B, C, D are the subsets of $\mathbb{Z}$ given by,

$A = \{x \in \mathbb{Z} : 0 \leq x \leq 10\}$, $B = \{x \in \mathbb{Z} : 5 \leq x \leq 15\}$

$C = \{x \in \mathbb{Z} : x \geq 5\}$, $D = \{x \in \mathbb{Z} : x \leq 15\}$ then find $A \cup B, A \cap B, B - C, A - D$.

(ii) Let p, q and r be statements. Then show that De Morgan's law

$\sim(p \vee q) = (\sim p) \wedge (\sim q)$ holds by truth table. (2+2+1+1)+4=10

(b) (i) If R be an equivalence relation on the set A, then show that R^{-1} is also an equivalence relation on *A*.

(ii) If $a \equiv b \ (mod\ m) and\ C \equiv d (mod\ m)$ then prove that $a + c \equiv (b + d)(mod\ m)$ and $ac \equiv bd(mod\ m)$. (2+2+2)+(2+2)=10

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B.Sc. Semester III (Honours) Examination, 2018-19

MATHEMATICS

Course ID : 32111 **Course Code : SHMTH-301C-5(T)**

Course Title : Theory of Real Functions and Introduction to Metric Space

Time: 2 Hours **Full Marks: 40**

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

1. Answer *any five* questions: 2×5=10

(a) If $\lim_{x\to a} f(x) = l$, then show that $\lim_{x\to a}|f(x)| = |l|$.

(b) Show that the function $f\colon [0,1] \to \mathbb{R}$ defined by

$f(x) = 1$, if x is rational

$= 0$, if x is irrational

is continuous nowhere in [0, 1]

(c) Prove that the function $f(x) = \frac{1}{x}, x \in (0,1]$ is not uniformly continuous on (0, 1].

(d) Is Rolle's theorem satisfied for $f(x) = x^2$ in [–1, 1]. Justify your answer.

(e) Show that $\log(1 + x) < x$ for $x > 0$.

(f) Expand $\tan^{-1} x$ (up to three terms).

(g) Give an example of a function f which satisfies the intermediate-value property on a closed and bounded interval $[a, b]$ but is not continuous on $[a, b]$.

(h) The function $d\colon R \times R \to R$ defined by $d(x, y) = |x - y|, \forall x, y \in \mathbb{R}$. Show that d is a metric on the set $\mathbb{R}$.

2. Answer *any four* questions: 5×4=20

(a) (i) If $f\colon S \to \mathbb{R}$ be differentiable at $c \in S$, then show that there exist $\delta > 0$ and a positive constant M such that $|f(x) - f(c)| \leq M\,|x - c|, \forall\, x \in S \cap N_\delta(c)$.

(ii) A function f is differentiable on [0, 2] and $f(0) = 0, f(1) = 2, f(2) = 1$. Prove that $f'(c) = 0$ for some c in (0, 2). 3+2=5

(b) Let $f\colon \mathbb{R} \to \mathbb{R}$ be a function satisfying $f(x + y) = f(x) + f(y), \forall\, x, y \in \mathbb{R}$. Show that if f is continuous at $x = a$, then f is continuous for all $x \in \mathbb{R}$. 5

(c) State and prove Cauchy's mean value theorem and hence deduce Lagrange's mean value theorem. 1+3+1=5

(d) Obtain Maclaurin's infinite series expansion of $\log(1+x)\,, -1 < x \le 1$. 5

(e) Examine the function $(x-3)^5(x+1)^4$ for extreme values. 5

(f) Let $(X,\ d)$ be a metric space and A° denote the interior of A. Then show that $(A \cap B)^\circ = A^\circ \cap B^\circ$. Is $(A \cup B)^\circ = A^\circ \cup B^\circ$ true for any subjects A, B of X? Give reason for your answer. 3+2=5

3. Answer *any one* question: 10×1=10

(a) (i) If $f(x)$ is a function defined on a deleted neighbourhood D of a point 'a' such that $f(x) \ge 0\ \forall\ x \in D$, then show that $\lim_{x \to a} f(x) \ge 0$, provided it exists.

(ii) If a function 'f' is continuous in a closed interval $[a, b]$, then show that 'f' is bounded in $[a, b]$. Also show that the converse of the above is not true in general.

(iii) Let $(X,\ d)$ be a metric space such that X contains more than one point and $A \subseteq X$. Show that a point $x \in X$ is a limit point of A iff every open sphere $S(x.r)$ contains infinitely many points of A. 3+(2+2)+3=10

(b) (i) Give an example of a pair of functions f, g on $[a, b]$ such that both f, g are discontinuous on $[a, b]$, but $f + g$ is continuous on $[a, b]$.

(ii) Show that the function $f(x) = \frac{1}{x^2}$ is uniformly continuous on $[a, \infty)$, where $a > 0$; but not uniformly continuous on $(0, \infty)$.

(iii) State and prove Taylor's theorem with Lagrange's form of remainder. 2+3+5=10

B.Sc. Semester III (Honours) Examination, 2018-19

MATHEMATICS

Course ID : 32112 **Course Code : SHMTH-302C-6(T)**

Course Title : Group Theory I

Time: 2 Hours **Full Marks: 40**

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

1. Answer *any five* questions: 2×5=10

(a) Let $G = \left\{\begin{pmatrix} a & 0 \\ b & 1 \end{pmatrix} : a, b \in \mathbb{R}, a \neq 0\right\}$. Does G form a group with respect to usual matrix multiplication? Justify your answer.

(b) Prove that the subgroup $SL_n(\mathbb{R})$ is normal in the general linear group $GL_n(\mathbb{R})$.

(c) Find the subgroup generated by $\{6, 8\}$ in the group $(\mathbb{Z}, +)$.

(d) Suppose G is a cyclic group such that G has exactly three subgroups viz. G, $\{e\}$ and a subgroup of order 7. What is the order of G?

(e) Find the order of the permutation $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 4 & 5 & 1 & 3 & 2 & 6 \end{pmatrix}$.

(f) Determine the number of elements of order 5 in $\mathbb{Z}_{25} \oplus \mathbb{Z}_5$.

(g) Let G be a group and $a \in G$. Prove that the mapping $f_a: G \to G$, defined by $f_a(x) = ax$ for all $x \in G$, is one to one and onto.

(h) Let G and H be two groups. Define a function $f: G \times H \to G$ by $f\big((a, b)\big) = a$ for all $(a, b) \in G \times H$. Prove that f is a homomorphism. Evaluate ker f.

2. Answer *any four* questions: 5×4=20

(a) Let H be a subset of a group G and let the set $N(H)$, called the normalizer of H in G, be defined by $N(H) = \{a \in G | aHa^{-1} = H\}$. Prove that $N(H)$ is a subgroup of G. If in addition H be a subgroup of G, then prove that H is normal in G if and only if $N(H) = G$. 2+3=5

(b) (i) Suppose H is a finite subgroup of a group G with $O(H) = n$. If there is no other subgroup of G with n elements then prove that H is a normal subgroup of G.

(ii) Prove that alternating group A_n is a normal subgroup of the permutation group S_n. 3+2=5

(c) Let, H and K be subgroups of a group G with K normal in G. Then prove that $H/H \cap K \simeq HK/K$. 5

(d) (i) Prove that $(\mathbb{Q}, +)$ is not cyclic.

(ii) Show that every subgroup of a cyclic group is cyclic. 2+3=5

(e) (i) Let G be a group of finite order n and $a \in G$. Then prove that $\text{o}(a)$ divides n and $a^n = e$.

(ii) Let p be a prime integer and a be an integer such that p does not divide a. Then using Lagrange's theorem show that $a^{p-1} \equiv 1(mod\ p)$. 2+3=5

(f) State and prove Cayley's theorem. 1+4=5

3. Answer *any one* question: 10×1=10

(a) (i) Show that any non-identity permutation $\alpha \in s_n (n \geq 2)$ can be expressed as a product of disjoint cycles, each of length ≥ 2.

(ii) Give an example (with reason) of a non-cyclic, commutative group of which each subgroup is cyclic.

(iii) If the order of a cyclic group G is divisible by a positive integer m then prove that there exists a unique subgroup of order m of G. 5+2+3=10

(b) (i) State and prove Lagrange's theorem on order of subgroup.

(ii) Show that $(\mathbb{Z}, +)$ and $(\mathbb{Q}, +)$ are not isomorphic but of same cardinality.

(iii) Find all the group homomorphisms from $(\mathbb{Z}_6, +)$ into $(\mathbb{Z}_4, +)$. 4+3+3=10

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